9

THE MOTION OF GASES IN THE SUN'S ATMOSPHERE

PART III. ON THE STRATIFICATION OF THE SOLAR ENVELOPE

BY A. K. DAS

(*Received for publication, November 18, 1940*)

ABSTRACT. In the introductory part of the paper the relative merits of existing theories of the solar chromosphere and their inadequacy to explain some of the salient facts of observation are discussed and the necessity for analysing the phenomena of the solar envelope from a different angle is emphasized. The dynamics of the motion of gases in the sun's atmosphere developed in an earlier paper is then used to evolve a mode of formation and maintenance of the different layers of the solar envelope. It is shown that this new process admits of quantitative treatment and yields heights of the reversing layer, the chromosphere and the quiescent prominences in good agreement with observation. The fact that all gases reach practically the same height in prominences, the increase of rotation speed with altitude in prominences, the fibrous structure of the chromosphere and several other facts of observation follow naturally from this process. The picture is however admittedly incomplete, but its success so far indicates that the dynamical principle on which it is based probably plays an important part in the phenomena of the solar envelope.

INTRODUCTION

The idea of selective radiation pressure advanced by Prof. M. N. Saha[1] in 1919 has been widely used for the interpretation of several astrophysical phenomena. Saha himself applied it to explain the occurrence, in the spectra of the early B-classes of stars, of the so-called "stationary" absorption lines H and K; he suggested that under the influence of selective radiation pressure singly ionised atoms of calcium were driven far away from these hot stars and formed a highly attenuated atmosphere which did not take part in the orbital motion of the stars, so that the stationary H and K lines were formed through the blanketing effect of this absorbing atmosphere of calcium vapour on the spectrum of the parent stars. This explanation must at present be regarded as superseded as the result of the long series of most careful observations made by Dr. J. S. Plaskett and his collaborators and in view of Sir A. S. Eddington's masterly theory of interstellar matter, both of which leave no reasonable doubt that diffuse matter containing calcium, sodium and perhaps most of the other elements present on the earth is uniformly distributed in interstellar space with a density of the order of 10^{-25} gm./cm.^{3}, except where local condensations form the diffuse nebulae. Nevertheless the idea of selective radiation pressure holds the field and has found numerous fruitful applications in recent astrophysical theories, notably those of the extended envelopes of Be stars and planetary nebulae. But the researches in which the most extensive and persistent use has been made of selective radiation pressure, in spite of considerable difficulties, are the theories of the solar chromosphere, which are mostly modifications of E. A. Milne's[2] theory of the calcium chromosphere, supported by light pressure. The mechanism of support of the CaII atom in Milne's theory is essentially the same as that indicated by Saha several years earlier, the principal feature of which is that the resonance line of CaII occurs in a part of the spectrum where the Sun radiates strongly, so that CaII atoms are subject to considerable radiation pressure which balances or sometimes even overbalances gravity. Milne has worked out the consequences of this idea at great length for the case of static equilibrium and has successfully accounted for the formation of a chromosphere of ionised calcium, of prominences of ionised calcium and for the occurrence of enormous velocities sometimes observed in eruptive prominences photographed in calcium light. Notwithstanding these successes, Milne's theory is confronted with considerable difficulties which have not so far been eliminated by the numerous attempts at modification and improvement made by S. R. Pike, J. Voltjer, S. Chandrasekhar[3] and others.

One of the principal difficulties of Milne's theory is that it cannot satisfactorily account for the permanent presence of hydrogen and helium in the chromosphere and in the prominences so long as the theory is based on the usual assumption that the intensity in the extreme ultraviolet solar spectrum follows a Planck law corresponding to a temperature of the order of 6000°K. The resonance lines of hydrogen are situated far outside the main region of the solar spectrum and in the case of helium the frequencies of the resonance lines are about twice as large as in the case of hydrogen; in fact, assuming that the Sun radiates as a black body at an effective temperature of 6000°K, the radiation pressure on hydrogen, according to Gurney, works out to be 10^5 times smaller than on singly ionised calcium and on helium 10^{13} times smaller. Yet it is a common fact of observation that all the three elements reach almost the same heights in the chromosphere and in the prominences. Besides, as D. H. Menzel[4] has pointed out, the quiescent type of equilibrium discussed by Milne is non-existent in the chromosphere. The evidence of eclipse observers shows that the chromosphere, like the corona, has a fibrous structure which is very different from that to be expected in an atmosphere approximately in hydrostatic equilibrium of the type tacitly assumed in Milne's theory. A formal possibility of escape from the discrepancy between the chromosphere of Milne's theory and the actual chromosphere has however been provided by Chandrasekhar, who has introduced a periodic space-term in the photospheric flux and has also taken account of the darkening towards the limb. Nevertheless Milne's theory in Chandrasekhar's modified form is far from being able to give a satisfactory explanation of some of the noteworthy features of the chromosphere, particularly the abundance of hydrogen. It seems fairly obvious that any theory which postulates the paramount importance of selective radiation pressure as a means of support for the chromospheric gases will be confronted with the extreme difficulty of explaining the existence of the major constituent, hydrogen, provided that the extreme ultraviolet spectrum of the Sun is taken to be identical with that of a black body at 6000°K. So forcibly has this difficulty impressed itself on some of the most eminent theoretical astrophysicists that S. Rosseland[5] says, "it may be wise to relax a little one's confidence in the significance of light pressure also as regards the distribution of calcium and allied elements in the chromosphere, pending the outcome of the case of hydrogen, helium and other elements with unfavourable resonance lines in the chromosphere and the prominences." He remarks further, "it is surely premature as yet to assert that we know enough about the stars to assert that light pressure is the only agency which can lead to the expulsion of particles. I think, it is fair to say that the evidence of solar observations, taken as a whole, is against this idea, and to such a degree that it fully justifies an attempt at the development of a non-committal formal analysis of the solar phenomena from a wider point of view." Rosseland himself has worked out a theory in which an envelope is kept distended through the bombardment by corpuscular rays (electrons) emitted by a star; with the help of this theory he has been able to explain some of the characteristic features of the solar chromosphere and the corona without recourse to the idea of support by radiation pressure. This theory has many obvious advantages, but it has not yet reached the stage of development at which a final judgment can be formed about it. Another theory which does not depend at least explicitly on the idea of selective radiation pressure has been advanced by W. H. McCrea[6] according to whom the chromosphere is supported by "turbulent motion." But McCrea defines this turbulence in a way quite different from what is understood by this term in hydrodynamics and bases his theory on the obviously untenable idea that the particles taking part in the turbulent motion have a Maxwellian velocity of the order of 15 km./sec., but not the temperature corresponding to this velocity. In spite of the unsatisfactory character of McCrea's theory, turbulence ought to play a part in the mechanics of the chromosphere, just as it cannot be denied that selective radiation pressure must have its contribution in the complex phenomena of the solar envelope though perhaps not in the manner postulated by Milne. In the present state of our knowledge, as McCrea has remarked, "we cannot say even whether radiation pressure provides the normal means of chromospheric support, with hydrogen and helium occupying somewhat exceptional positions, or whether, on the other hand, normal chromospheric support comes from quite a different agency with a few gases like CaII and SrII showing additional peculiarities owing to their happening to be subject in addition to radiation pressure." In these circumstances the following attempt at exploring an alternative mode of formation of the solar envelope, somewhat different from those hitherto considered and yet incorporating some of their features, may not be entirely superfluous.

MECHANICS OF EJECTION OF SOLAR GASES

As mentioned in the foregoing section, the evidence of experienced eclipse observers is quite convincingly in favour of the view that the solar envelope is to be looked upon not as a true atmosphere like that of the earth but as composed of innumerable jets of glowing gases emanating from the body of the Sun. The appearance of the chromosphere is that of a tangle of interlacing trajectories which, though not so pronounced, is very similar to that of the corona; also the agglomeration of matter in the solar envelope is very many times more than what is to be expected on any simple hydrostatic theory. The chromosphere, the corona and the prominences would therefore seem to be due to a complex of dynamical phenomena in which there may be no static equilibrium. Indeed it seems most probable that there is no essential difference between the modes of occurrence of these varied phenomena, so that the division of the envelope into several strata may merely represent different degrees of the same underlying process. Whatever may be this underlying mechanism it must possess certain properties which are indicated by facts of observation. For instance, it must be capable of explaining why no essential differences of prominence structure are noted in photographs of prominences taken in not only the strong lines of hydrogen, helium, ionised calcium, ionised strontium and ionised titanium, but also in the lines of faint and medium intensities. This observation imposes an important requirement upon the mechanism of formation of prominences, namely, that the acceleration acting upon the various types of atoms must be at least approximately equal. This condition cannot be satisfied by selective radiation pressure so long as its effect is negligibly small on hydrogen ; for it seems scarcely reasonable to expect that the most abundant constituent can be dragged along by the comparatively rare atoms of ionised calcium, etc., which may develop considerable velocities through selective light pressure.

The assumption that the Sun's spectrum in the extreme ultraviolet is exactly the same as that of a black body at 6000°K has however, as Saha [7] has recently argued, no unquestionable experimental or theoretical justification. In fact, Saha has urged that certain features of the night-sky spectrum and of the ionospheric phenomena require that the unobservable part of the ultraviolet spectrum of the Sun should consist of a faint continuous background on which are superposed intense emission lines of H, He^+, Fe^+ and other elements which are represented in the visible range by lines of the subordinate series. Should this hypothesis represent the actual spectrum of the Sun, the difficulties about the support of chromospheric and prominence gases against gravity would be considerably reduced. In fact, K. O. Kiepenheuer[8] has argued that the extreme ultraviolet spectrum of the Sun must be as postulated by Saha and that an exact compensation of gravity in the quiescent prominences and in the chromosphere actually takes place through selective radiation pressure arising out of the absorption of L_a radiation by the H-atoms which are by far the most abundant constituents of the solar envelope. It may be remarked in this connection that the emission lines in the extreme ultraviolet solar spectrum postulated in Saha's hypothesis arise through the mechanism of fluorescence in an attenuated atmosphere as suggested by Rosseland in a famous paper in 1926; if the compensation of gravity in the chromosphere takes place in the way suggested by Kiepenheuer then the mechanism of fluorescence must operate in some layer between the photosphere and the chromosphere. If this intermediate layer is identified with the reversing layer or some transition layer between the photosphere and the reversing layer, then in this layer the conditions of pressure (which according to St. John and Babcock's scheme is at least 10^{-2} atms. and probably much higher) would be unfavourable for the operation of Rosseland's process of fluorescence and consequently the chromosphere could not be effectively supported by selective radiation pressure acting on hydrogen. If, on the other hand, the pressure of the photospheric gases is less than 10^{-1} atm. and the mean presssure in the reversing layer is at most 10^{-4} atm. as concluded by Russell and Stewart, then the reversing layer, the chromosphere and the prominences may all be supported by selective radiation pressure acting on hydrogen, provided the Sun radiates in the way suggested by Saha. This mode of support of the solar envelope is apparently not entirely free from uncertainty. There is however an alternative; the Sun emits continuously high velocity electrons for which there is direct evidence; the bombardment of the atoms of the envelope by these electrons may compensate the effect of gravity on these atoms and may conceivably fulfil also the functions attributed to the absorption of L_a radiation. But, whatever the ultimate explanation may be, several facts of observation show that the effect of gravity on the solar envelope is completely or almost completely neutralised by some mechanism which at present remains obscure.

In almost all the current theories of the chromosphere, the atoms forming the chromosphere are assumed to have originated in the photosphere or just above it. But, if we accept the view that there is probably no essential difference between the processes underlying the formation of a prominence and of the chromosphere, then there is good reason for thinking that the chromospheric gases have their origin below the photosphere and probably in the deep interior of the Sun; for there is little doubt that the quiescent prominences originate in the interior of the Sun. In fact, starting from the generalised postulate that all the material of the solar envelope comes from the interior of the Sun, one can arrive at a way of visualising the complex phenomena of the solar envelope which seems to have distinct advantages over the pictures which form the basis of the current theories of the chromosphere. For this purpose we have first to consider the process through which a mass of material can be ejected from the interior and then the motion of this mass in the space above the photosphere.

According to the current theory of internal constitution, the temperatures within the stars are of the order of a few million degrees and at these temperatures the radiation consists of waves of about the same lengths as X-rays, while matter is broken up mostly into atomic neuclei and electrons. The conditions in the interior of stars are almost those of an ideal thermodynamical enclosure, so that all particles behave like black bodies and the radiation pressure is the same as the light pressure calculated from Maxwell's electromagnetic theory of light. Under equilibrium conditions at any point in the interior gravity is balanced by the combined effect of gas pressure and radiation pressure. This state of static equilibrium can be maintained also in a rotating star so long as the rotation is like that of a rigid body and the rate of generation of energy at every point of the interior depends only upon the density and the temperature in the way required by a well-known theorem due to H. v. Zeipel. But the theories of Jeans,[9] Eddington[10] and Rosseland[11] show that an actual star does not rotate like a rigid body; in fact it is generally accepted that the interior of a star rotates much faster then the surface and that the angular speed of rotation increases rapidly with the depth below the surface. According to Eddington an actual star does not obey v. Zeipel's theorem and this leads to local variations of temperature in the interior. Thus, from theoretical considerations, localised rises of temperature in the interior of the Sun are to be expected, but we can draw this inference also from the fact that on the Sun's surface sudden localised increases of temperature (eruptions) are often observed. It may be mentioned in this connection that Eddington has shown that the non-fulfilment of v. Zeipel's theorem results in the establishment of a system of internal circulatory currents; Rosseland has concluded that there is a possibility of these internal currents being periodically unstable. It seems likely therefore that the periodic variation of solar activity is intimately connected with the periodic instability of the internal currents. But the fairly common occurrence of eruptive phenomena on the Sun's surface may be regarded as indicating that internal disturbances do occur also frequently and quite irregularly.

Let us suppose that in a limited region of the Sun's interior the temperature rises suddenly to a value higher than the normal. This increase of temperature will give rise to increased radiation pressure and a consequent outflow of matter in a radial direction. Since the radiation pressure experienced by the constituent particles is due to the amount of the forward momentum of the radiation absorbed or scattered by the particles the velocity of ejection will depend upon the cross-sections and the weights of the particles as well as on the rate of recombination of the neuclei and electrons. In the absence of precise knowledge about these factors it is not possible to estimate with any certainty the initial velocity of ejection of the particles for a given increase of radiation. But without any calculations it can be safely concluded that at the pressure prevailing in the interior of the Sun there will be frequent collisions among the various types of particles resulting in a re-distribution of the acquired momenta and an eventual movement of the whole mass of matter affected by the disturbance. The velocity with which the mass of gases will move outwards towards the photosphere will of course depend upon the actual increase of radiation pressure and since the radiation pressure is proportional to the fourth power of the temperature it is likely that appreciable velocities will be acquired through comparatively small rises of temperature. The velocity with which a mass of gas thus ejected from the interior will emerge from the photosphere will, because of frictional resistance, be considerably less than the initial velocity which the mass started with. Outside the photosphere the frictional resistance will be small and we may ignore it as a rough approximation. If, in addition, gravity is neutralised by some such process as discussed earlier in this paper, then the magnitude of the velocity with which the mass of gas emerges from the photosphere will remain unchanged during the subsequent motion. Now, since the ejected mass originated in the interior of the Sun where the angular speed of rotation is much faster than at the surface, it will tend to rotate with a higher speed than the surface of the Sun. This brings us precisely to the hypothesis which forms the basis of the considerations set forth in Part I.[12] Recently I have come to know that an almost identical hypothesis was put forward in 1935 by Mr. J. Evershed [13] in order to explain the discrepancy between the angular motion of prominences derived by him from his measures of Doppler shift by the spectrographic method and the rotation speed derived from the times of successive meridian passages of prominences and dark markings.* For comparison I quote Mr. Evershed's hypothesis in his own words. "A possible explanation may now be suggested of the increasing angular speed with height above the photosphere, and of the decreasing speed with decreasing solar activity. This depends upon the fact, I believe fairly well established, that the gases above the photosphere do not form a true atmosphere to the Sun, but consist of innumerable jets of luminous gas projected radiallly outwards from beneath the photospheric level. If the interior of the Sun is rotating faster than the photosphere, and with uniform angular speed in all latitudes, then these jets will tend to retain the velocity of the interior regions, and the deeper the origin of a jet the more closely will its motion conform to the angular speed of the interior. The prominences coming from the deepest layers will give the greatest rotation speeds, and the most uniform angular motion. At times of intense solar activity the great prominences, streaming out at definite points in

* I am indebted to Messrs. P.R.C. Iyer and B. G. Narayan for drawing my attention to a paper by R. de Lury (Canada, R.A.S., November, 1939), which was received at the Kodaikanal Observatory a few months ago and in which the apparent slight increase of the rotation speed with altitude above the photosphere is explained by the hypothesis that the gases of the envelope originate in the interior of the Sun and retain the high angular speed of rotation of the interior. It is clear from the paper that the hypothesis is originally due to Mr. Evershed.

the photosphere, are probably ejected from greater depths than the smaller prominences seen at times of minimum activity." It is evident from the above quotation that the essential features of Mr. Evershed's hypothesis are the same as those of the hypothesis the dynamical consequences of which were worked out in Part I. From the results obtained in Part I it was evident that many of the major features of prominences and dark markings could be explained both qualitatively and quantitatively on the basis of the dynamical ideas developed there. From the details of the process of ejection of matter from the interior of the Sun just discussed and the equations of motion, as derived in Part I, of a mass of matter thus ejected in the space above the photosphere, it is clear that some of the peculiar features of the solar envelope discussed earlier in this paper bear a similarity to the peculiarities of the dynamical picture here presented.

As shown in Part I, the motion of a mass of gas (ejected from the interior of the Sun) in the space outside the photosphere will be controlled by the dynamical equations—

$$\left.\begin{aligned}
&\text{Equatorwards}: \quad m\frac{d^2x}{dt^2} = \mathrm{X} \quad 2m(\omega'-\omega)v\sin\phi + m(\omega'-\omega)^2\mathrm{R}\sin\phi\cos\phi \\
&\text{Westwards}: \quad m\frac{d^2y}{dt^2} = \mathrm{Y} + 2m(\omega'-\omega)(u\sin\phi + w\cos\phi) \\
&\text{Upwards}: \quad m\frac{d^2z}{dt^2} = \mathrm{Z} - 2m(\omega'-\omega)v\cos\phi + m(\omega'-\omega)^2\mathrm{R}\cos^2\phi.
\end{aligned}\right\} \qquad (12)$$

In these equations the positive x-axis points towards the equator, the positive y-axis towards the west and the positive z-axis points vertically upwards ; u, v, w are the components of the velocity of the moving mass (m) ; X, Y, Z are components of the external forces including gravity, friction, etc.; R is the radius of the Sun, ϕ the heliographic latitude, ω the angular velocity at the surface of the Sun and ω' the higher angular velocity of the interior retained by the ejected mass of gas. If the mass, as it emerges from the photosphere, has only a velocity V directed vertically upwards and has no external forces acting on it, then the equations (12) can be written

$$\left.\begin{aligned}
\frac{d^2x}{dt^2} &= -2(\omega'-\omega)v\sin\phi + (\omega'-\omega)^2\mathrm{R}\sin\phi.\cos\phi \\
\frac{d^2y}{dt^2} &= 2(\omega'-\omega)(u\sin\phi + w\cos\phi) \\
\frac{d^2z}{dt^2} &= -2(\omega'-\omega)v\cos\phi.
\end{aligned}\right\} \quad \dots \quad (12\cdot1)$$

Now from equations (12·1) it is clear, since at the initial moment $u=v=0$, $w=\mathrm{V}$, that the velocity u arises out of the acceleration d^2x/dt^2 which must be of a smaller order than d^2y/dt^2 and d^2z/dt^2 and therefore we may also, as rough approximation, neglect u against w and v. The equations (12·1) may then be simplified into

$$\left.\begin{array}{ll} \text{Equatorwards :} & \dfrac{d^2x}{dt^2} = 0 \\[2ex] \text{Westwards :} & \dfrac{d^2y}{dt^2} = 2(\omega' - \omega)w \cos\phi \\[2ex] \text{Upwards :} & \dfrac{d^2z}{dt^2} = -2(\omega' - \omega)v \cos\phi. \end{array}\right\} \quad \ldots \quad (12{\cdot}2)$$

The second equation shows that if the particles of the gas mass have an *upward velocity*, they are acted upon by a *westward horizontal* acceleration which deflects them from their vertical path. On the other hand, if the particles have a *downward velocity* the path is deflected *eastward*. Similarly the third equation shows that if the particles have an *eastward velocity*, they are acted upon by an *upward deflective acceleration* and, conversely, a *westward velocity* gives rise to a *downward deflective acceleration*. It is to be noted that the deflective accelerations depend only on the velocity, so that if the particles started with the same initial velocity they would maintain their relative positions throughout the subsequent motion and the trajectories of the different particles would be identical. Under the combined action of the equations (12·2), a particle moving vertically upwards at the start will travel in a direction more and more inclined to the west of the vertical until the velocity becomes entirely horizontal and westward ; thereafter the particle will turn progressively downward and finally the velocity will become entirely horizontal but directed eastward so that the particle must rise again. In the idealised condition of absence of friction and other external forces the particle will go on moving indefinitely along a trajectory which will consist of a series of overlapping loops. In reality, however, the motion cannot be frictionless and it will also be complicated by the equatorward acceleration which will not be nil. The actual motion of gases in the solar envelope will be a three-dimensional spiral motion of great complexity and will resemble highly turbulent motion. Nevertheless our idealised equations of motion may be used for drawing general conclusions which should be reliable so far as the orders of magnitude are concerned.

DIVISION OF ENVELOPE INTO LAYERS

A glance at equations (12·2) shows that a particle describing a periodic trajectory in accordance with these equations reaches a maximum height (above the photosphere) which depends on the initial vertical velocity (V) at the moment of emergence from the photosphere and on ϕ and $\omega' - \omega$. For a given value of ϕ and of $\omega' - \omega$ therefore all particles emerging from the photosphere with a given vertical velocity will reach the same height and the natural upper boundary of the trajectories of all such particles will be a spherical surface situated at this height above the photosphere. It seems possible that in this way different boundary layers in the envelope may be formed by particles emerging with different ranges of velocity. In order to test this possibility quantitatively we have to calculate the maximum height reached by particles with a given velocity of emergence (V). Integrating the third of equations (12·2) with the initial conditions $t=0$, $w=V$ and $v=0$, we have

$$z = -(\omega' - \omega)v \cos\phi \cdot t^2 + Vt.$$

Now, at the highest point of the trajectory $w=0$ and $v=V$, so that the average value of v during the upward motion is $\frac{1}{2}V$. Then the maximum height reached by a particle is given by

$$z_{max} = -\tfrac{1}{2}(\omega' - \omega)V.\cos\phi.t^2 + Vt. \quad \ldots \quad (12{\cdot}3)$$

Similarly, integrating the second of equations (12·2) and remembering that the average value of w during the ascent is $\frac{1}{2}V$, we get

$$y_{max} = \tfrac{1}{2}(\omega' - \omega)V.\cos\phi.t^2. \quad \ldots \quad (12{\cdot}4)$$

Eliminating t from (12·3) with the help of (12·4), we have

$$z_{max} = -y_{max} + \sqrt{\frac{2y_{max}.V}{(\omega' - \omega)\cos\phi}}. \quad \ldots \quad (12{\cdot}5)$$

Now, as in Part I, we identify y_{max}/z_{max} as the tangent of the tilt of the trajectory from the vertical. Then $\tan\delta = y_{max}/z_{max}$. In Part II[14] we have shown that the average tilt (δ) of a prominence is of the order of 8° as determined from the measurement of a large number of dark markings. Assuming that this is generally valid, we put $y_{max} = \tan 8°$. $z_{max} = 0.14\ z_{max}$ in (12·5) and obtain

$$z_{max} = 0{\cdot}216.\frac{V}{(\omega' - \omega)\cos\phi}. \quad \ldots \quad (12{\cdot}6)$$

If the particles are ejected from all depths of the interior, we may on the average put, as in Part I, for the equator ($\phi = 0$) $\omega' = 10\omega$, so that $\omega' - \omega = 2{\cdot}63 \times 10^{-5}$ and we have

$$z_{max} = 0{\cdot}082 \times 10^5.V. \quad \ldots \quad (12{\cdot}7)$$

Now, if the reversing layer, the chromosphere and the stable prominences are formed in the purely dynamical way as contemplated in the present paper, then the values of z_{max} computed from (12·7) by using appropriate values of V derived from observation should agree with the observed heights of the prominences, the chromosphere and the reversing layer so far as the orders of magnitude are concerned. Unfortunately there is considerable uncertainty about the observed values of V. According to St. John[15] the iron lines, belonging to the lowest level, give an upward velocity of about 0.2 km./sec. We may take this as the value of V for the reversing layer. St. John also finds that the calcium vapour, to which the bright emission line K_2 is due, has an ascending motion over the general surface of the sun of 1.97 km./sec. in the mean. We use this as the value of V for the chromosphere. St. John's determinations are from spectrographic measurements of Doppler shifts and therefore possess a much higher degree of accuracy than the values of upward velocity available in the case of the prominences. According to Pettit[16] the commonest velocities observed in the internal motions of a typical stable prominence are 5-10 km./sec. We therefore take 7.5 km./sec. as the value of V in an average prominence of the stable type. Using these values of V we obtain from (12·7) the following values for the heights of the tops of the different layers above the photosphere :

Reversing Layer	...	1640 km.
Chromosphere	...	16150 km.
Average Prominence	...	61300 km.

It is to be noted in this connection that according to observational practice the height of a prominence is reckoned from a level about 10″ above the photosphere and not from the photosphere. In order to compare with the observational height we have therefore to subtract about 8000 km. from the above calculated height of an average prominence. For ready comparison between the observational values and the values calculated above the following table has been prepared. The observed values of the heights of the reversing layer and the chromosphere are taken from St. John and Babcock's[17] scheme of the Sun's atmosphere and the observed height of a typical quiescent prominence is taken from Abetti—*The Sun*, 1938, p. 142.

Part of Envelope	Height (observed)	Height (calculated)
Reversing Layer ...	1500 km.	1640 km
Chromosphere ...	14000 km.	16150 km.
Prominence ...	50000 km.	53300 km.

It is evident from the table that the agreement between observation and theory is very satisfactory indeed. Although, in view of the various approximations used in the calculations, one should not place excessive confidence in numerical agreements, yet it seems safe to conclude from the results obtained that the dynamical process considered in the present series of papers ought to play a fundamental part in the formation of the solar envelope. The corona has been purposely left out of our discussion as the equations (12) are valid so long as it is justified to neglect x, y and z against R (*cf.* Part 1). Although these equations ought to be applicable to the inner corona, they cannot have application in the mechanics of the outer corona which extends to several times the solar radius.

CONCLUSIONS

In the above calculations we have put on the average $\omega'-\omega=9\omega$ at the equator on the supposition that the particles are ejected from all depths in the interior ; and by using values of V derived from observation we have found that the calculated heights of an average stable prominence, the chromosphere and the reversing layer agree with the observed heights. Now, from equation (12.6) we see that these heights are proportional to the ratio $V/(\omega'-\omega)$, so that we can always obtain the same heights for any arbitrary values of V provided we adjust the values of $\omega'-\omega$ suitably. But observational data show that only particular values of V occur in the quiescent prominences, the chromosphere and the reversing layer ; and our calculations have shown that with these values of V we have to use a particular value of $\omega'-\omega$ at the equator, namely $\omega'-\omega=9\omega$ in order to obtain good agreement between the observed and calculated heights. From this we may draw the conclusion that the matter which mostly forms the quiescent prominences, the chromosphere and the reversing layer originates not at all depths but only at a particular depth at which the angular speed at the equator is such that the ejected matter has an angular speed about 10 times the angular speed of the photosphere. It may be noted that this depth roughly corresponds to the core which the sun is believed to possess. Thus, from the view-point here advocated, the core is the seat of the disturbances which lead to the ejection of matter from the interior of the Sun. This constitutes an important difference between our hypothesis and Mr. Evershed's hypothesis quoted earlier in this paper. While, according to Evershed, the increase of angular speed with height above the photosphere is to be explained by supposing that the gases constituting the higher levels of the envelope originate in the deeper levels of the interior (having higher angular speeds), according to our point of view the gases which form the different levels of the envelope originate in the same internal level (the core), but gases of the higher levels of the envelope have been ejected with greater velocities and therefore shew an apparent higher rotation speed. This would of course lead us to expect an increasing angular speed and also increasing heights of the prominences, the the chromosphere and the reversing layer with increasing solar activity; such variations appear actually to happen except in the case of the reversing layer for which there seems to be no observation available in this respect.

Our dynamical picture reduces the origins of the reversing layer, the chromosphere and the stable prominences to one and the same general process and brings out clearly the possibility that there is no fundamental difference between the modes of formation of these phenomena. According to this picture ordinary stable prominences are formed by gases ejected from limited regions of the Sun's interior and emerging from the photosphere with radial velocities ranging from 2 to 7.5 km./sec. roughly ; such a prominence must be continually supplied with matter ejected from the disturbed region in the interior as long as the prominence lasts. The chromosphere and the reversing layer are formed by particles ejected from the interior and emerging from the photosphere with radial velocities ranging respectively from 0.2 to 2 km./sec. and from 0 to 0.2 km./sec. roughly ; as these layers are permanent features of the Sun, they have to be constantly supplied with matter from widespread and permanently active regions in the interior. According to this view there is no static equilibrium in the quiescent prominences, the chromosphere and in the reversing layer ; the equilibrium is dynamic. All these conclusions appear to harmonise with observational facts. Our picture is, however, far from being complete ; it will require much further development, particularly the part that concerns the interior of the Sun, before it can attempt to answer the questions as to why particles of definite ranges of velocity emerge from the photosphere and why the vertical density-gradient in the envelope is exponential ; but even as it stands at present it seems to point the way in which the answers to such questions are to be sought.

Before concluding we may mention an indefiniteness in our calculations which arises out of the uncertainty in the nature of the variation of the angular velocity (ω') with latitude in the interior of the Sun. There are two extreme ways in which ω' may vary with latitude : one is that ω' increases with latitude proportionally to $\sec^2\phi$ for which there is theoretical justification ; the other is that ω' decreases with increasing latitude in the same way as ω on the surface is observed to vary. In Part I the first type of variation of ω' was used in calculating the variation of the meridian inclination of dark markings with latitude and excellent agreement was obtained between the calculated and the observed values. If this type of variation of ω' is used in (12 6) above, then the heights of stable prominences, the chromosphere and the reversing layer should decrease with increasing latitude becoming zero at the pole. Observation however shows that there is no evidence of this kind of variation of height; in fact, the chromosphere appears to have practically the same height at all latitudes. If we use the second type of variation of ω' with latitude in (12·6), then it is evident that the heights of prominences, the chromosphere and the reversing layer would increase with increasing latitude becoming infinite at the pole, which is again contrary to observation. On this basis the calculated variation of the meridian inclination of dark markings will still show the same trend as the observed variation, but the agreement between the calculated and the observed curves will not be so good as before. The actual variation of ω' with latitude may however be something in between the two extreme types which will give a satisfactory agreement between observation and theory both as regards the latitude variation of meridian inclination of dark markings and the heights of prominences, the chromosphere and the reversing layer.

ADDENDUM

In calculating the heights of the reversing layer, the chromosphere and an average stable prominence we have taken y_{max}/z_{max} to be equal to the tangent of the average tilt of the ascending part of the trajectory. Also in a stable prominence the gases are continually going up and dissipating at the top, so that what we see as a prominence is mainly the ascending portion with some of the top part of the trajectory; accordingly in evaluating z_{max} we have put y_{max}/z_{max} equal to the tangent of the observed average tilt of stable prominences. This procedure may not be strictly correct, but it is not a bad approximation; besides it has the advantage that it indirectly takes account of the retarding effect of frictional resistance and incomplete compensation of gravity on the velocity of ejection (V). One may however evaluate z_{max} by first finding the general solution of equations (12·2) and then making allowances for the effect of retarding forces on V. As this procedure is more general and permits of the evaluation of z and y at any point of the trajectory, it seems desirable to include this also here. We put $2(\omega' - \omega) \cos \phi = b$ and write equations (12·2) as follows:

$$d^2z/dt^2 = -b.dy/dt \quad \ldots \quad (I)$$

$$d^2y/dt^2 = b.dz/dt. \quad \ldots \quad (II)$$

In order to solve these equations we put $p = dz/dt$; then we have

$$d^2y/dt^2 = b.p;$$

$$dy/dt = b.p.t, \text{ since for } t=0,\ dy/dt=0;$$

$$d^2z/dt^2 = -b^2pt = dp/dt;$$

$$d^2p/dt^2 = -b^2p. \quad \ldots \quad (III)$$

The general solution of (III) is

$$p = C_1 \sin bt + C_2 \cos bt,$$

where C_1 and C_2 are constants. Now $dz/dt = V$ when $t=0$; therefore $C_2 = V$, and $dz/dt = C_1 \sin bt + V.\cos bt$. ... (IV)

Integrating (IV) with the initial conditions $t=0$, $z=0$, we have

$$z = \frac{V}{b}.\sin bt + \frac{C_1}{b}.(1-\cos bt).$$

From (II) and (IV) we have with the initial conditions $t=0$, $y=0$, $dy/dt=0$

$$d^2y/dt^2 = b(C_1 \sin bt + V \cos bt),$$

$$dy/dt = -C_1 \cos bt + V \sin bt + C_1, \quad \ldots \quad (V)$$

$$y = C_1\left(t - \frac{\sin bt}{b}\right) + \frac{V}{b}\cdot(1-\cos bt).$$

Evaluating C_1 in the usual way with the help of (V), (IV) and (I), we have $C_1 = 0$ and therefore finally we get

$$\left.\begin{aligned} z &= \frac{V}{b}\cdot \sin bt \\ y &= \frac{V}{b}\left(1-\cos bt\right) \end{aligned}\right\} \quad \ldots \quad (VI)$$

Now z is maximum when $\sin bt$ is equal to 1; therefore we have

$$z_{max} = 0.5 \cdot \frac{V}{(\omega' - \omega) \cos \phi}. \qquad \ldots \quad \text{(VII)}$$

This is the same relation as (12.6) obtained previously except for the numerical factor. It is to be noted that the derivation of (VII) implies that V remains constant all along the trajectory so far as its magnitude is concerned. In reality however the magnitude of **V** will vary through the influence of retarding forces, but it is uncertain by what factor it is actually reduced. It is clear, however, that a factor of about 0.4 would make (VII) the same as (12·6). St. John (Ap. J., Vol. 32, p. 36, 1910) finds that towards the base of the chromosphere the velocity is about 2 km./sec. upwards and near the top of the chromosphere it is about 1 km./sec. downwards. This observation makes a factor of 0.4 entirely likely, so that by the alternative method of calculation considered here we would obtain the same heights for the reversing layer, the chromosphere and an average stable prominence as obtained previously. These heights would be the same at all latitudes if the latitude variation of ω' is such that $\omega' - \omega$ varies as $\sec \phi$, so that $(\omega' - \omega) \cos \phi$ is a constant.

Incidentally it is to be noted that the relations (VI) may be expected to have a bearing on the forms of active prominences and their movement towards apparent centres of attraction. I hope to discuss these questions in a later paper.

KODAIKANAL,
S. INDIA.

R E F E R E N C E S

1 M. N. Saha, *Ap. J.*, September (1919).

2 E. A. Milne, *M.N.R.A.S.*, **86**, 578 (1926), and later papers.

3 S. Chandrasekhar, *M N.R.A.S.*, **94**, 14 and 726 (1933-34).

4 D. H. Menzel, *Pub. Lick Observatory*, **17**, I (1931).

5 S. Rosseland, *Pub. Univ. Observatory*, Oslo, **5** (1933); Theoretical Astrophysics, Oxford (1936).

6 W. H. McCrea, *M.N.R.A.S.*, **89**, 718 (1929) and **95**, 80 (1934).

7 M. N. Saha, *Proc. Roy. Soc.* (*A*), **160**, 155 (1937).

8 K. O. Kiepenheuer, *Zeit F. Astroph.*, Bd. **15**, S. **53** (1938).

9 J. H. Jeans, *M N R.A.S*, **86**, 328 and 444 (1926).

10 A. S. Eddington, *M.N.R.A.S.*, **90**, 54 (1929).

11 S. Rosseland, *Astrophysica Norvegica*, II, **3** (1936).

12 A. K. Das, *Ind Journ. Physics*, **14**, V, 369-86 (1940).

13 J. Evershed, *M N.R.A S.*, **95**, 507 (1935)

14 A. K. Das and B G. Narayan, *Ind Journ Physics*, **15** 17 (1941).

15 St. John, *Ap. J*, **32**, 36 (1910); also see Abetti—*The Sun*, 190 (1938) and Abbot, *The Sun*, 100-102 (1929).

16 E. Pettit, *Ap. J.*, **76**, 42 (1932).

17 St. John and Babcock, *Ap. J.*, **60**, 406 (1924).